1 a

$$\frac{1}{\sqrt{2}-3} = \frac{1}{\sqrt{2}-3} \times \frac{\sqrt{2}+3}{\sqrt{2}+3}$$
$$= -\frac{\sqrt{2}+3}{7}$$

b

$$\frac{3}{\sqrt{5}-1} = \frac{3}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$$
$$= \frac{3(\sqrt{5}+1)}{4}$$

c

$$\frac{2}{2\sqrt{2}-1} = \frac{2}{2\sqrt{2}-1} \times \frac{2\sqrt{2}+1}{2\sqrt{2}+1}$$
$$= \frac{4\sqrt{2}+2}{7}$$

d

$$\frac{3}{\sqrt{5}-\sqrt{3}} = \frac{3}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$
$$= \frac{3(\sqrt{5}+\sqrt{3})}{2}$$

e

$$\frac{1}{\sqrt{7}-\sqrt{2}} = \frac{1}{\sqrt{7}-\sqrt{2}} \times \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}$$
$$= \frac{\sqrt{7}+\sqrt{2}}{5}$$

f

$$\frac{1}{2\sqrt{5}-\sqrt{3}} = \frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$$
$$= \frac{2\sqrt{5}+\sqrt{3}}{17}$$

2 a

$$x^2+x-1=0 \Leftrightarrow x = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$x^2+bx+1=0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2-4}}{2}$$

Case1

$$-b+\sqrt{b^2-4} = -1+\sqrt{5}$$
$$1-b = \sqrt{5}-\sqrt{b^2-4}$$
$$1-2b+b^2 = 5-2\sqrt{5(b^2-4)}+b^2-4$$
$$-2b = -2\sqrt{5(b^2-4)}$$
$$b^2 = 5(b^2-4)$$
$$20 = 4b^2$$
$$b = \pm\sqrt{5}$$

Case2

$$-b+\sqrt{b^2-4} = -1-\sqrt{5}$$
$$1-b = -\sqrt{5}-\sqrt{b^2-4}$$
$$1-2b+b^2 = 5+2\sqrt{5(b^2-4)}+b^2-4$$
$$-2b = +2\sqrt{5(b^2-4)}$$
$$b^2 = 5(b^2-4)$$
$$20 = 4b^2$$
$$b = \pm\sqrt{5}$$

Comment: Because of squaring solutions should be checked

b **i** If $b = \sqrt{5}$, solutions of $x^2 + bx + 1 = 0$ are:

$$x = \frac{-\sqrt{5} \pm \sqrt{5-4}}{2}$$

$$x = \frac{-\sqrt{5} \pm 1}{2}$$

The common solution is $\frac{-1-\sqrt{5}}{2}$

ii If $b = -\sqrt{5}$, solutions of $x^2 + bx + 1 = 0$ are:

$$x = \frac{\sqrt{5} \pm \sqrt{5-4}}{2}$$

$$x = \frac{\sqrt{5} \pm 1}{2}$$

The common solution is $\frac{-1+\sqrt{5}}{2}$

3 $n^2 - 6n - 7 = a + bn + cn^2 - cn$

Equating coefficients

$c = 1, a = -7, b - c = -6$

$\therefore a = -7, b = -5, c = 1$

4 $a = k_1 n$ and $b = k_2 n$

$\therefore a - b = k_1 n - k_2 n = (k_1 - k_2)n$

5 **a** $576 = 2^6 \times 3^2$,

$\sqrt{576} = 2^3 \times 3 = 24$

b $1225 = 5^2 \times 7^2$,

$\sqrt{1225} = 5 \times 7 = 35$

c $1936 = 4^2 \times 11^2$,

$\sqrt{1936} = 4 \times 11 = 44$

d $1296 = 6^4$,

$\sqrt{1296} = 6^2 = 36$

6

$$\frac{x+b}{x-c} = 1 - \frac{x}{x-c}$$

$$x + b = x - c - x$$

$$x = -b - c$$

7

$$\frac{1}{x-a} + \frac{1}{x-b} = \frac{2}{x}$$

$$x(x-b) + x(x-a) = 2(x-a)(x-b)$$

$$x^2 - bx + x^2 - ax = 2x^2 - 2(a+b)x + 2ab$$

$$-(a+b)x = -2(a+b)x + 2ab$$

$$x = \frac{2ab}{a+b}$$

8 One solution is $x = 5, y = 14$

Therefore $x = 5 + 12t, y = 14 - 5t, t \in \mathbb{Z}$

For positive $t > -\frac{5}{14}$ and $t < \frac{14}{5}$

That is $t = 0, 1$ or 2.

The other two solutions are $(17, 9)$ and $(29, 4)$

9 Let n be the number of books bought at $\$25$. Let m be the number of books bought at $\$35$.

$25n + 35m = 190$

$\therefore 55n + 7m = 38$

One solution is $n = 2$ and $m = 4$. There are no other solutions.

10 $x^2 - 4x - 8 - \lambda(x^2 - 2x - 5)$

$= a(x^2 - 2bx + b^2)$

Equating coefficients

$x^2 : 1 + \lambda = a \ldots (1)$

$x : -4 - 2\lambda = -2ab \ldots (2)$

Constant : $-8 - 5\lambda = ab^2 \ldots (3)$

Substitute from (1) in (2) and (3)

$-4 - 2\lambda = -2b(1 + \lambda) \ldots (4)$

$-8 - 5\lambda = (1 + \lambda)b^2 \ldots (5)$

From (4), $b = \dfrac{2 + \lambda}{1 + \lambda}$

Substitute in (5)

$-8 - 5\lambda = \dfrac{(2 + \lambda)^2}{1 + \lambda}$

$(-8 - 5\lambda)(1 + \lambda) = (2 + \lambda)^2$

$\lambda = -\dfrac{3}{2}$ or $\lambda = -\dfrac{4}{3}$

Find $a = -\dfrac{1}{3},\ b = -2,\ \lambda = -\dfrac{4}{3}$;

$a = -\dfrac{1}{2},\ b = -1,\ \lambda = -\dfrac{3}{2}$

11a $5, 1$

b $\dfrac{8}{3}, 0$

c $3, -\dfrac{3}{5}$

d $14, -6$

e $1, 9$

f $4, -\dfrac{4}{3}$

g $\dfrac{5}{2}, -\dfrac{15}{2}$

12a $\{x : -2 \leq x \leq 2\}$

b $\{x : x \leq -1\} \cup \{x : x \geq 1\}$

c $\left\{x : \dfrac{1}{2} \leq x \leq \dfrac{9}{2}\right\}$

d $\{x : -1 < x < 2\}$

e $\left\{x : x \leq -\dfrac{1}{2}\right\} \cup \left\{x : x \geq \dfrac{7}{2}\right\}$

f $\left\{x : -\dfrac{1}{3} \leq x \leq \dfrac{5}{3}\right\}$

13a $x = \dfrac{(y - 3)^2 + 1}{2}$

b $x = \frac{1}{3}\left(\frac{4}{(y+2)^2} - 1\right)$

14 150 minutes

15a $x = \frac{51}{25},\ y = \frac{32}{25}$

b $x = \frac{a(b^2+1)}{a^2+b^2},\ y = \frac{b(a^2-1)}{a^2+b^2}$

16a 3

b 12

c 8

17a $\Delta = 4a(a-1)$

b **i** $a = 1$

ii $a > 1$ or $a < 0$

iii $0 < a < 1$